1. Consider a linear time invariant system with input and output through the equation:

\[ y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(t-2) d\tau \]

(a) What is the impulse response \( h(t) \) for this system? (10%)

(b) Is the system BIBO stable? Why? (10%)

2. Consider a linear time invariant system for which we are given the following information:

\[ X(s) = \frac{s^2 + 2}{s - 2}; \quad x(t) = 0, t > 0 \quad \text{and} \quad y(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t) \]

(a) Determine the transfer function \( H(s) \) of the system. (10%)

(b) Using \( H(s) \) found in part (a), determine the output \( y(t) \) if the input is \( x(t) = e^{3t}, -\infty < t < \infty \). (5%)

3. Suppose the closed-loop poles of a feedback system satisfy the following equations:

(a) \[ \frac{1}{(s + 2)(s + 3)} = -\frac{1}{k} \]; (b) \[ \frac{s - 1}{(s + 1)(s + 2)} = -\frac{1}{k}, k > 0 \].

Using the root-locus method to determine the values of \( k \) for which the feedback system is guaranteed to be stable. (30%)

4. Consider a feedback control system with the characteristic equation:

\[ 1 + k \frac{s + 1}{s(s - 1)(s + 6)} = 0 \]

From Routh’s criterion, what is the range of \( k \) for which this system is stable. (15%)

5. A state space model for a linear system is given by

\[ \dot{X}(t) = \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \]

\[ y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} X(t) \]

(a) Find the transfer function of the system. (10%)

(b) Determine the Observability and the controllability of the system. (10%)